7.2.2 MOTION—SPEED, VELOCITY AND ACCELERATION[M32]

We have seen that kinetic energy is directly related to the motion of an object. We need now to look more closely at motion itself, to examine how objects move and what it is that makes them move.

7.2.2.1 The Nature of Motion

The study of motion is fundamentally concerned with changes in position. When something has changed its position with respect to its surroundings, we say that it has moved. Ultimately, we will also be interested in measuring how quickly an object changes position or moves.

If you see a car in front of your house and later see it farther along the street, you are correct in assuming that the car has moved. To reach this conclusion, you observed two positions of the car and you also noted the passage of time. You might not know how the car got from one position to the other. It might have moved at a steady rate or it might have speeded up and then slowed down before it got to its second position.

It might also have been moving when you first noticed it and might still have been moving at its second location. The car may have moved from one location to the other in a straight line, or it may not have. But none of these possibilities changes the truth of the statement that the car has moved.

Usually, when we see something move, we do not just make two observations. We observe the moving object continuously. Even a case of continuous observation, however, can be thought of as a series of observations in which the interval of time between two successive observations is small.

Carry out Experiment 32.1 and discuss the relationship between movement and distance between the dots on the ticker tape.

We can make a statement about the position of an object at each moment in time, by stating position as a function of time. It can be helpful to represent such functions graphically. For example, suppose that an object can move only back and forth along a single line, like a train on a track, but is at rest 2 metres from the origin. The algebraic model, or description, of this is d=2 metres, where d is used to represent its distance from the origin. The graphical model is illustrated in Figure 7.2.2.1. Note the coordinates of this graph. The value of d is plotted as a function of time, t. In this case, d always has the same value regardless of time.

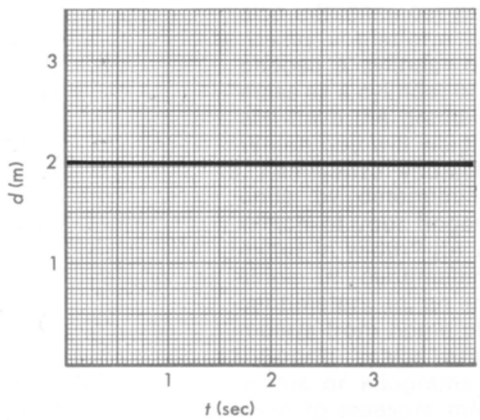


Figure 7.2.2.1 Graph of d = 2 m

A slightly more complicated case is that in which the object is moving at a steady rate so that for every second that passes it is 2 metres further to the right. If we start our clock just as the object passes the origin, we can describe the functional dependence of position *d* on time *t* with the equation:

$$d \text{ metres} = 2 \text{ metres/second} \times t \text{ seconds}$$

Note how the graph (Figure 7.2.2.2) corresponds with the equation for $t = 0, 1, 2, 3$ seconds.

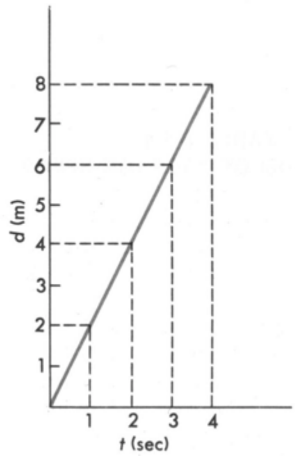


Figure 7.2.2.2
Graph of d = 2t

Carry out Experiment 32.1a and discuss the relationship between speed/velocity and the distance between the dots on the ticker tape.

Figure 7.2.2.3 illustrates the graph of a still more complicated motion. During the first second, $d = 1$ m, indicating that the object is at rest 1 m from the origin. In the next three seconds, its position gradually increases from 1 m to 4 m, so that it is travelling at a speed of 1 m/s. In the next second it moves a further 4 m, at 4 m/s, before coming to rest for one second, 8 m from the origin.

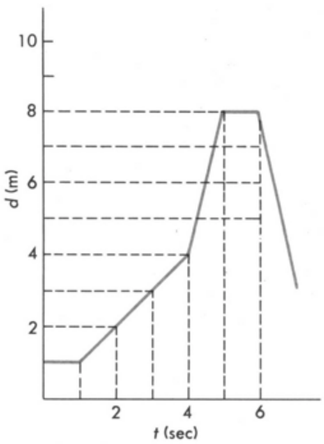


Figure 7.2.2.3
Motion Comprising Several Segments

A change of position in a particular direction is called a **displacement**. A displacement has magnitude, direction, and a point of origin. A displacement is always a straight line segment from one point to another point, even though the actual path of the moving object between the two points might be curved. Furthermore, displacements are vector quantities and can be combined like other vector quantities.

For example, the statement that an aeroplane flew 500 kilometres north from Sydney describes the aeroplane's displacement, since the statement specifies the point of origin, magnitude, and direction. Whether the aeroplane flew in a straight line or not, the aeroplane's displacement vector is a straight arrow directed north from Sydney and representing a magnitude of 500 kilometres.

7.2.2.2 Scalars and Vectors

Physical quantities such as length, area, volume, mass, density and time can be expressed in terms of magnitude alone, as single numbers with suitable units. The length of a table, for example, can be completely described as 1.5 m. The mass of a steel block could be completely described as 54 kg. Quantities, such as these, that can be expressed completely by single numbers with appropriate units are called scalar[1] quantities, or simply **scalars**.

Other physical quantities, such as displacement, velocity, force, acceleration, electric field strength, and magnetic induction, cannot be completely described in terms of

[1] The word "scalar" is derived from the Latin *scala*, meaning "ladder" or "steps", which implies magnitude.

magnitude alone. In addition to magnitude these quantities always have a specific direction. Quantities that require magnitude and direction for their complete description are called vector[2] quantities They can be represent by **vectors** and their behaviour can be described by certain mathematical rules.

We often use the words speed and velocity interchangeably when referring to how quickly something is moving. Speed and velocity, however, are not the same in the scientific context. We will see that speed can be completely described in terms of magnitude alone, and is thus a scalar quantity. Velocity, however, must be described in terms of both magnitude *and* direction, and is thus a vector quantity. Similarly, distance and displacement are related scalar and vector quantities respectively—distance can be completely described in terms of magnitude, while displacement requires both magnitude and direction. Note, however, that in neither case is the scalar quantity (speed or distance) necessarily simply the magnitude of the related vector quantity (velocity or displacement).

A vector is usually denoted in print by bold face type, *e.g.* ***V***. A vector can be denoted conveniently in handwriting by underscoring the letter ($\underline{V}$) or by putting an arrow over it ($\vec{V}$). The magnitude of a vector is indicated in ordinary type: thus *V* denotes the magnitude of the vector ***V***. the magnitude of a vector is always taken as a *positive* quantity. A negative sign before the symbol indicating a vector merely changes the sense of the direction—it interchanges the head (arrow tip) and tail without changing the length or orientation of the line segment.

7.2.2.2.1 Vector Analysis

The graphical representation of a vector is an arrow whose length is proportional to the magnitude of the vector and whose direction is that of the vector. A vector diagram is a scale drawing of the various velocities, forces, or other vector quantities involved in, for example, the motion of a body. The sum of several vectors is the single vector leading from the tail of the first vector to the head of the last vector when the vectors are placed head to tail, in any order, with their lengths and original directions kept unchanged (D in Figure 7.2.2.4). This process of summing vectors is called vector addition.

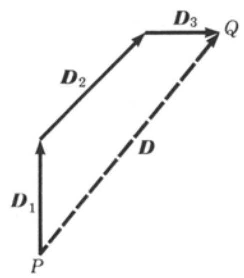
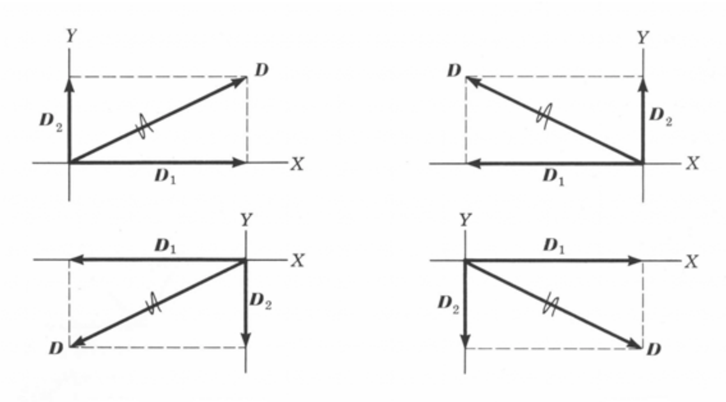


Figure 7.2.2.4
Vector Addition

Demonstrate, on a piece of graph paper, that the sum of several vectors is the same, regardless of the order in which the vectors are placed.

A vector can also be resolved into two or more other vectors whose sum is equal to the original vector. The new vectors are called the **components** of the original vector, and are normally chosen to be perpendicular to one another (Figure 7.2.2.5).

[2] The word "vector" is derived from the Latin word meaning "carrier", which implies displacement.



In the illustration, $\boldsymbol{D_1}$ and $\boldsymbol{D_2}$ are the components of the vector $\boldsymbol{D}$ along the x and y axes respectively.

Figure 7.2.2.5 Vector Components

In the reverse process, where two components are *resolved* into a single vector, the new vector is called the **resultant**. Note, however, that this is just a special case of vector addition.

7.2.2.2.2 Typical Applications of Vector Analysis

We will see that vector analysis is fundamental to the resolution of many problems involving physical quantities such as velocity and force.

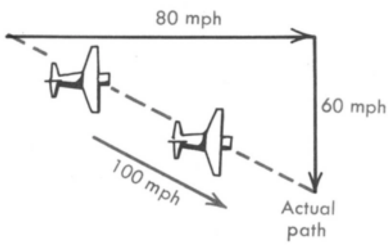


Figure 7.2.2.6 The pilot keeps a constant due east heading while being carried south

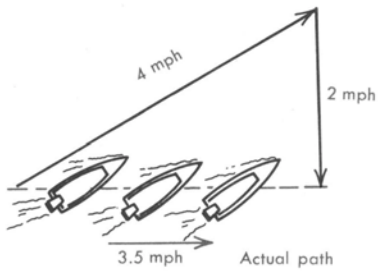


Figure 7.2.2.7 The boat maintains a due east direction by heading into the current

7.2.2.3 Speed

Speed is the rate of change of position, expressed as a distance moved in unit time. The SI units for speed are metres per second (m/s or m·s^{-1}).

7.2.2.3.1 Uniform Speed

If we represent speeds graphically, the lines that represent greater speeds are more steeply inclined, *i.e.* they have a greater slope. The slope of a line is the ratio of its vertical component to the corresponding horizontal component and thus, the slope of line **A** in Figure 7.2.2.8 is

$$\frac{3.0}{1.0} = 3.0$$

The speed represented by line **A** is thus 3.0 m/s. Similarly, the speeds represented by lines **B** and **C**, with decreasing slopes, are 1 m/s and 0.33 m/s respectively.

These lines are all representations of constant, or uniform speed—the slope, and hence speed, is constant throughout.

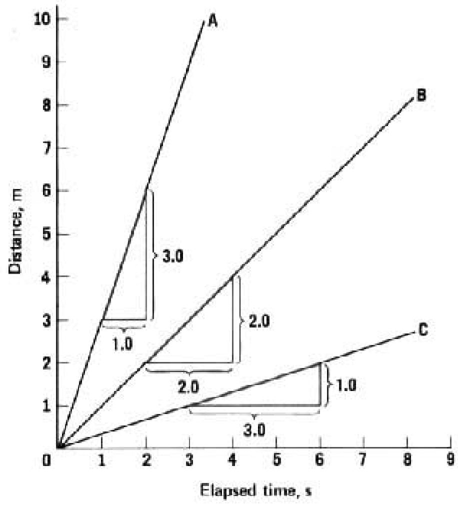


Figure 7.2.2.8
Three graphs of motion with different constant speeds. **A** is for 3 m/s, **B** is for 1 m/s and **C** is for 0.33 m/s.

Carry out Experiments 32.2 & 32.3.

7.2.2.3.2 Average Speed

The graph representing the speed of an object may not, however, be a straight line. The motion represented by the graph in Figure 7.2.2.3 above is an example of motion with non-uniform speed. More typically, however, an object changes speed steadily, and its motion is represented by a curved line.

The motion of a car (see Figure 7.2.2.9), travelling between two corners of a street block is illustrated graphically in Figure 7.2.2.10.

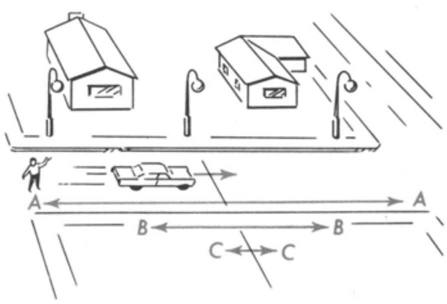


Figure 7.2.2.9
Measuring the speed of a car

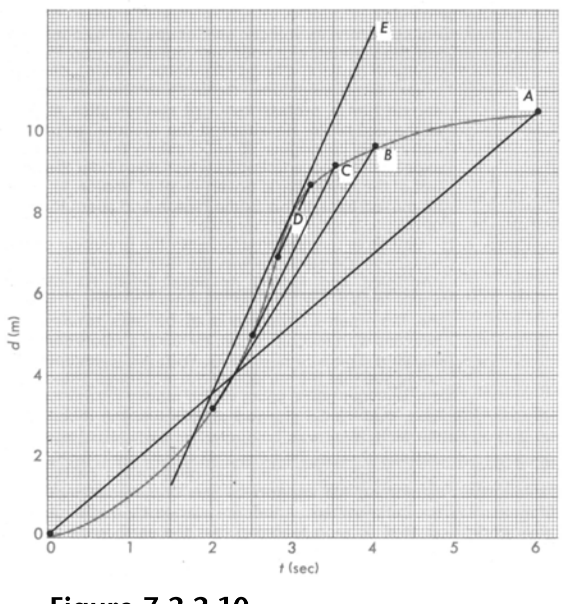


Figure 7.2.2.10
Graph of Displacement *vs* Time

In such cases it will often be useful to refer to the *average speed* of the object. Average speed is found by dividing the total distance travelled by the elapsed time.

$$\textbf{Average Speed} = \frac{\textit{distance travelled}}{\textit{elapsed time}}$$

In the example represented in Figure 7.2.2.10, the car travels 10.4 metres in 6 seconds, so the average speed will be:

$$Average\ Speed = \frac{10.4\ metres}{6\ seconds} = 1.73\ m/s$$

While this is the average speed, the car will sometimes be travelling more slowly than this, and sometimes more quickly.

Note that for an object moving at a uniform speed, however, the average speed will always be equal to the speed of the object.

7.2.2.3.3 Instantaneous Speed

In addition the average speed, we will often wish to know the speed of an object at a particular point in time—the instantaneous speed. Without a speedometer on an object, it is difficult to determine its instantaneous speed. For practical purposes, however, instantaneous speed can be measured approximately by timing motion over a short distance. This method gives only an approximation of the instantaneous speed—it is actually the average speed over the time interval used.

When dealing with uniform motion this is not an issue, as the instantaneous speed is always same—the speed of the object or the slope of the line in its graphical representation. When dealing with motion such as that represented by the graph in Figure 7.2.2.10, things are a little more complicated—instantaneous speed is the slope of the line that is *tangent* to the curve at a given point.

We can see how the average speed over a short distance or time interval serves as an approximation for the instantaneous speed if we look at the lines in Figure 7.2.2.10. The problem here is to find the instantaneous speed after 3 seconds (*i.e.* when t = 3). An initial estimate might simply be the average speed over the entire trip, this being represented by the slope of line **A**. Line B, however, represents the average speed over an interval of just 2 seconds, rather than the whole 6, so we might reasonably expect this to give a more accurate value of the instantaneous speed. Similarly, lines **C** and **D** give successively more accurate values as the time interval gets shorter, as shown in the table below.

Table 7.2.2.1 Average Speed for Decreasing Time Interval

Line	Δd	Δt	$\bar{v}$
A	10.4	6.0	1.73
B	6.4	2.0	3.2
C	4.2	1.0	4.2
D	1.8	0.4	4.5

These figures indicate that the line **D** is a reasonable estimate, since the speed it yields does not differ greatly from that derived from the line **C**. Although not shown in the illustration, an interval of 0.1 sec also yields a value of 4.5 m/s. When we get to an interval so small that making it still smaller would not change the value obtained for the speed, we have the instantaneous speed.

Line **E** is the tangent to the curve at t = 3.0 sec. When we study calculus, we will see that the slope of this line yields the instantaneous speed. In the present case, we can see that line **D** is essentially parallel to this tangent, and is thus a good approximation.

7.2.2.4 Velocity

When both speed and direction are specified for the motion, the term velocity is used—velocity is speed in a particular direction. Displacement is a measure of position in a given direction relative to some point of reference. Thus, velocity can also be defined as the rate of change of displacement in unit time.

The SI units for velocity are metres per second (m/s or $m{\cdot}s^{-1}$).

A person walking eastward does not have the same velocity as a person walking northward, even if their speeds are the same. Both people might travel with a *speed* of 5 km/h, but one will have a *velocity* of 5 km/h to the east and the other a *velocity* of 5 km/h to the north. Of course, two people also have different velocities if they walk in the same direction at different speeds.

The instantaneous velocity of an object may change in direction as well as in magnitude, but the rule for finding the average velocity is essentially the same as that for finding the average speed:

$$\boldsymbol{v} = \frac{\boldsymbol{d}}{t}$$

where $\boldsymbol{d}$ is the displacement from the starting point,
t is the time taken to reach the terminal point, and
using the direction from the starting point to the terminal point
(see Figure 7.2.2.11).

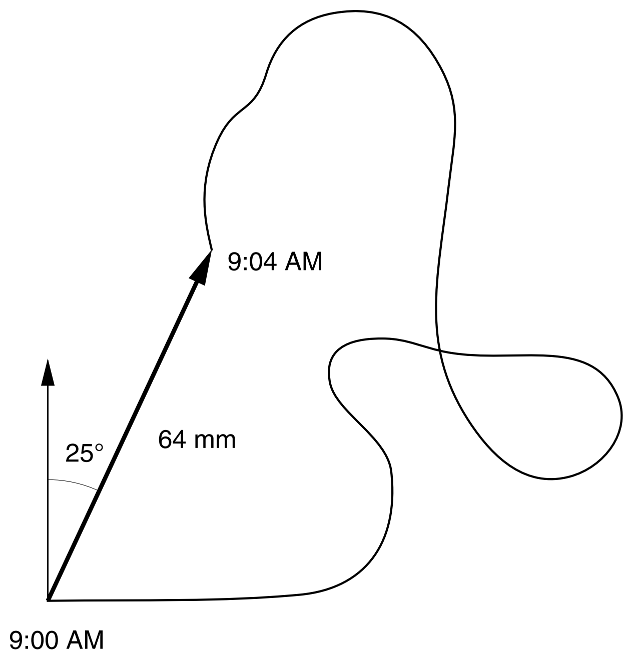


Figure 7.2.2.11
The path of a caterpillar on a sheet of graph paper. The average velocity of the caterpillar is 15 mm/min at an angle of 25° from the reference direction.

7.2.2.5 Acceleration

A body whose velocity is not constant is said to be accelerating. Acceleration is defined as the time rate change of velocity and is thus a measure of how quickly the velocity is changing. Acceleration, like velocity, is a vector quantity so this term is applied whether the velocity is increasing, decreasing or changing in direction.

Carry out Experiment 32.4.

7.2.2.5.1 The Nature of Acceleration

An object accelerates when it is acted on by an unbalanced force.

The most obvious illustration of acceleration is provided by a motor car. A car has an accelerator pedal which, when depressed, makes the car go faster or increase in speed. This is what most people think of immediately when we mention acceleration—an increase in speed, or *positive* acceleration. What might not be immediately obvious is that the brake pedal is also used to induce acceleration. The brakes are used to make a car slow down, or decrease in speed. This *negative* acceleration is more commonly referred to as deceleration or retardation.

Since acceleration is a vector quantity, it can also be induced by a change of direction, even if the speed of the object in question remains constant. For the time being, however, we will restrict ourselves to the discussion of acceleration of objects travelling in a straight line.

For an object starting at rest and travelling in a straight line, its acceleration is given by the equation:

$$\boldsymbol{a} = \frac{\boldsymbol{v}}{t}$$

where $\boldsymbol{v}$ is the velocity, and
t is the time taken to reach this velocity.

As an example, a car that goes from rest (0 km/h) to 60 km/h in l0 seconds accelerates at a rate of:

$$\frac{60\ {}^{km}\!/_{h}}{10\ s} = 6.0\left({}^{km}\!/_{h}\right)/_{s}$$

This will be a greater acceleration than that of a car that goes from rest (0 km/h) to 60 km/h in 15 seconds:

$$\frac{60\ {}^{km}\!/_{h}}{15\ s} = 4.0\left({}^{km}\!/_{h}\right)/_{s}$$

The SI units of acceleration are metres per second per second, or metres per second squared (m/s^2 or $m{\cdot}s^{-2}$).

7.2.2.6 Equations of Motion

Displacement (**s**), velocity (**v**), acceleration (**a**) and time (t) are related by four general equations of motion, which apply when acceleration is constant and motion is constrained to a straight line.

For an object moving with uniform acceleration in a straight line, its velocity at any point in time is given by the equation:

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t \qquad \textbf{(1)}$$

where **v** is the velocity;
u is the initial velocity (the velocity when the acceleration commenced);
a is the acceleration; and
t is time over which the object is accelerated.

Further, the displacement of such an object is given by the equation:

$$\mathbf{s} = \frac{(\mathbf{v} + \mathbf{u})t}{2} \qquad \textbf{(2)}$$

where **s** is the displacement; and
v, **u** & t are as defined above.

Thus, the distance travelled by an object moving with uniform acceleration in a straight line is given by the equation:

$$\mathbf{s} = \mathbf{u}t + {}^{1}\!/_{2}\mathbf{a}t^2 \qquad \textbf{(3)}$$

where **s** is the displacement; and
u, **a** & t are as defined above.

The relationship between displacement, velocity and acceleration is also then given by the equation:

$$\mathbf{v}^2 = \mathbf{u}^2 + 2\mathbf{as} \qquad \textbf{(4)}$$

where **v**, **u**, **a** & **s** are as defined above.

Look at graphs of **s** *vs* **t**, **v** *vs* **t** and **a** *vs* **t** for constant displacement, velocity and acceleration.

7.2.2.7 Dimensions of Quantities

We have set up standards for three quantities that are arbitrarily chosen as the basis of our measurement system: *length*, *mass* and *time*. Many other variables can be expressed in terms of just these three. It is also useful to consider variables in terms of *dimensions* related to these three basic quantities. Length, regardless of whether it is measured in metres, feet, furlongs or miles, has the dimension of a length—**L**. Mass, being another fundamental quantity, has its own dimension—**M**. Time also has its own dimension—**T**.

Velocity, as we have seen, is some length divided by a unit of time. All velocities thus have the dimension of **L/T**, regardless of whether we are talking about an average velocity, constant velocity, or instantaneous velocity, and regardless of whether it is specified in metres per second, miles per hour or furlongs per fortnight.

Any area must have the dimension of $\mathbf{L}^2$. This does not mean that we are talking only about a square. The sides of a rectangle are measured in units of length, each with dimension **L**, so that that when we multiply the lengths of the two sides together to yield the area, the two dimensions are also multiplied together to yield the dimension of the area: $\mathbf{L} \times \mathbf{L} = \mathbf{L}^2$. The area of a circle is πr^2, or the surface of a sphere $4\pi r^2$. In any area calculation there are exactly two lengths to be multiplied, although there may also be constants involved (π and 4π respectively in the previous examples).

A volume is always the product of three lengths, and thus has dimension $\mathbf{L^3}$. We recognise this when we express volumes in *cubic* metres. In a rectangular solid, this is the product of length, width and height. For a sphere, the volume is $4/3\pi r^3$.

The density of a substance is defined as its mass per unit volume, and thus has dimension $\mathbf{M/L^3}$.

7.2.2.7.1 Dimensional Analysis

When an equation is written relating one group of variables to another, it will always be necessary to have the same overall dimensions on one side of the equal sign as on the other. Consider the following example.

A car can travel at 60 miles per hour. To emphasise the dimension of velocity, we write this: 60 miles/hour. The *unit* used to measure *length* is the mile, and the *unit* used to measure *time* is the hour. The dimension of miles/hour is $\mathbf{L/T}$. Of course, we noted above that any velocity must have dimension $\mathbf{L/T}$, regardless of whether the units are miles per hour, metres per second, or furlongs per fortnight.

Suppose we want to change the units from miles per hour to metres per second. There are 1610 metres in a mile, so that:

$$\frac{1610\,\text{metres}}{1\,\text{mile}} = 1$$

Therefore, multiplying or dividing anything by this fraction will not change any value. Similarly:

$$\frac{1\,\text{hour}}{3600\,\text{seconds}} = 1$$

and we can also multiply by this. We can make the conversion by multiplying 60 miles/hour by one twice:

$$60\,\frac{\text{miles}}{\text{hour}} \times \frac{1610\,\text{metres}}{1\ \text{mile}} \times \frac{1\ \text{hour}}{3600\,\text{seconds}} = 27\,\frac{\text{metres}}{\text{second}}$$

The units can be treated like algebraic quantities and can be cancelled out leaving the desired final units. This system provides a test for correctness of the conversion.

The dimensions can also be treated as algebraic quantities. How long does it take the car going at 100 kph to travel 300 km?

$$\begin{aligned} \text{velocity} &= \frac{\text{distance}}{\text{time}} \\ \therefore\ \text{time} &= \frac{\text{distance}}{\text{velocity}} = \frac{300\,\text{km}}{100\,\text{km/hour}} \\ &= 3\,\text{hours} \end{aligned}$$

The dimensions for this equation are:

$$\mathbf{T} = \frac{\mathbf{L}}{\mathbf{L}/\mathbf{T}} = \frac{\mathbf{L}}{1} \times \frac{\mathbf{T}}{\mathbf{L}} = \mathbf{T}$$

since the $\mathbf{L}$ in the numerator cancels out the $\mathbf{L}$ in the denominator.